Classical Duality from Dimensional Reduction of Self Dual 4-form Maxwell Theory in 10 dimensions

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Abstract

By dimensional reduction of a self dual p-form theory on some compact space, we determine the duality generators of the gauge theory in 4 dimensions. In this picture duality is seen as a consequence of the geometry of the compact space. We describe the dimensional reduction of 10-dimensional self dual 4-form Maxwell theory to give a theory in 4-dimensions with scalar, one form and two form fields that all transform non trivially under duality.

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Introduction

Recently there has been much interest in field and string theory dualities. It has been shown that it is possible to explain some gauge theory dualities from a geometric point of view by considering a self dual theory living in $M^4 \times K$ where K is some compact manifold. When specifying the self duality condition it is necessary to break general covariance and specify a direction, chosen to be on the compact space. Different theories from a 4-dimensional point of view then become related to different directions on the compact space. However, the physics must be independent of which direction is chosen. Hence the duality becomes a consequence of general covariance in the self dual theory.

This elegant explanation of gauge duality has been applied in a variety of contexts. In several papers [1,2] various authors have shown how S-duality in Maxwell theory in 4-dimensions can be produced by considering the compactification of self dual two form theory in 6-dimensions. The rank of the form refers to the gauge potential, which for a $p + 1$ form theory couples to a p-brane via its world volume. Hence the 6-dimensional theory concerns a gauge theory coupling to self dual strings of which there has been much interest recently [3]. The case we will consider in detail is an extension to the normal ansatz for a self dual 4-form in 10 dimensions compactified on T^6. We will begin with a brief summary of the method used to derive the generators of the duality transformation for Maxwell theory in 4-d from the self dual 6-d theory. Then we will discuss the 10-dimensional case and our extension to the usual ansatz. The resulting theory in 4-d will be derived along with the generators of the duality group acting on the 4-dimension theory. The results may be interpreted in the context of type IIB string theory of which this is a part of the bosonic sector, where for small values of the string coupling constant the gravitational sector decouples. Indeed some authors have used a similar construction to investigate the duality properties of dyonic p-branes in the context of IIB supergravity [4,5]. We shall not explore such applications here.

S-duality from self dual 6-dimensional Maxwell Theory

Here we briefly review the work of Verlinde and Giveon et al [1,2] as this will provide the main method for constructing the duality generators in the more complex case considered later.

It has been shown that determining a covariant action for self dual p-form fields is nontrivial [6]. Recently, there has been some work demonstrating how this might be done [7]. However, following [1], we will describe the theory in a non covariant way via an action

for the ordinary Maxwell type theory with the self duality condition removed and then later impose the self duality constraint as an extra equation, not derived from the action. The action for a Maxwell theory in six dimensions is given by:

$$S = \int_{M^6} dB \wedge H + \frac{1}{2} \int_{M^6} H \wedge^* H, \tag{1}$$

where $H \in \Lambda^3(M^6)$ and $B \in \Lambda^2(M^6)$. We shall use the notation $\Lambda^p(M^d)$ for the space of p forms in the d dimensional manifold, M^d. To impose the self duality relation we use the non covariant equation:

$$i_v(H - dB) = 0, \tag{2}$$

where i_v denotes the inner product with a vector field v. We then dimensionally reduce the theory; $M^6 \rightarrow M^4 \times T^2$, including only the zero modes of fields on T^2. The ansatz for H and B is then as follows:

$$H = F_D a + F b$$

$$B = A_D a + A b \tag{3}$$

where $F_D, F \in \Lambda^2(M^4)$ and $A, A_D \in \Lambda^1(M^4)$ and $a, b \in \Lambda^1(T^2)$ As will be checked later this ansatz is consistent with the self duality in 6-dimensions. The a and b 1 forms on T^2 are the canonical closed 1 forms associated with the non trivial homology one cycles on the torus. Hence they form a basis for $H_1(T^2, \mathbf{Z})$. In the canonical basis the intersection matrix is such that:

$$\int_{T^2} a \wedge b = 1$$

Essentially this means the area of the torus is defined to be 1. The period matrix Π is given by:

$$\Pi = \int_{T^2} \begin{pmatrix} a \wedge^* a & a \wedge^* b \\ b \wedge^* a & b \wedge^* b \end{pmatrix} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix} \tag{4}$$

First, a direction for the vector field v must be specified so as to explicitly impose the self duality condition. We will choose v such that $i_v a = 0$. Inserting the ansatz (3) into (2) then gives the usual relation for an abelian field strength;

$$F = dA. \tag{5}$$

Later we will repeat the procedure with a different choice for v and generate the dual theory. The equivalence of the two duality related theories is based on the independence of any

physics on our choice of v. We then substitute (5) and (3) into the action (1) and obtain a 4-dimensional action after using (4) to do the integration over the compact space. This gives:

$$S_4 = \int_{M^4} dA \wedge F_D + \frac{1}{2} \int_{M^4} (F_D dA) \Pi \begin{pmatrix} F_D \\ dA \end{pmatrix}, \tag{6}$$

where we have thrown away an irrelevant total derivative. We are left with a theory that contains an auxillary field F_D that we may integrate out of the partition function. The integral is Gaussian in nature and so the integration is trivial. Of course it is that possible anomalous effects might appear in the determinant. As such, aspects of the global topology of the space time will be important. In references [1,8], the effects of the topology on the path integral are discussed. We shall not repeat their results here. The equations of motion for F_D give:

$$F_D = \tau_2^* dA - \tau_1 dA.$$

Hence F_D becomes identified with the dual field strength and the self duality in 6-dimensions of the ansatz (3) can be confirmed. After integrating out the F_D field we have the usual Maxwell theory in 4 dimensions with a theta term:

$$S_4 = \int_{M^4} \tau_2 F \wedge^* F + \tau_1 F \wedge F$$

We can then identify τ_1 and τ_2 with the theta term coupling and the usual electric charge coupling of the Maxwell theory. (Some overall factors of 2π are needed outside the action to be able to make direct comparison with the usual τ parameter.)

The generator of the S-duality is found by choosing the other possible direction for v in the choice of definition of self duality. Taking v such that $i_v b = 0$ and then repeating the calculation carried out above gives the following:

$$S_4 = \int_{M^4} \frac{\tau_2}{|\tau|^2} F \wedge^* F - \frac{\tau_1}{|\tau|^2} F \wedge F$$

Putting τ together as $\tau = \tau_1 + i\tau_2$ then we have the familiar S-duality of $\tau \to -1/\tau$. The other generator of SL(2,Z) $\tau \to \tau+1$ is found by considering the topological term $F \wedge F$. This has been well explored in the literature, see for example reference [8]. Geometrically, τ and 1 define the basis vectors for a lattice, L. The torus is then described by C^2/L. $\tau \to \tau+1$ and $\tau \to -1/\tau$ leave the torus invariant. Hence, it is possible to understand S-duality as a result of the geometry of some compactified space. The generators of the duality transformation are found by different choices of the direction by which self duality is described in the compactified space. This calculation has been generalized to any Riemannian Manifold and to higher dimensional

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compact spaces by various authors [2], including the case of $K^3 \times T^2$, which would be of particular interest from a string theory point of view. Dimensional reduction of self dual p-form gauge theories has been investigated recently in other contexts by [9,10].

Theories in higher dimensions

Crucial to the discussion above was the existence of a self dual 2 form gauge theory in 6 dimensions. As is well known [11], self dual (chiral) gauge theories only exist when the dimension of spacetime D= 2 mod 4. Hence for the next case we will look at D=10 self dual 4 form abelian gauge theory. This will also provide us with a far richer duality group since the compact manifold will be 6 dimensional.

First we will deal with the most immediate generalization of the above calculation. It is useful to consider the general context of how the p-form field strength is decomposed between the compact, D dimensional space, K^D and the, d dimensional space-time, N^d. Hence, in general

$$H \in \Lambda^{(D+d)/2}(M^{D+d})$$

Set $p = (D + d)/2$

$$\Lambda^p(M^{D+d}) = \Lambda^{d/2}(N^d) \otimes \Lambda^{D/2}(K^D) \bigoplus_i [\Lambda^i(N^d) \otimes \Lambda^{p-i}(K^D) \oplus \Lambda^{p-i}(N^d) \otimes \Lambda^i(K^D)]$$

Where i runs from 0 to $i \leq d$ and $i \leq D$ but not i such that $p - i = D/2$. The space has been split up according to the action of the Hodge operation on elements in the space. The first space in this decomposition is singled out because the Hodge operation maps an element in the space back into the same space. The other spaces in the direct sum appear in pairs with the Hodge operation mapping an element in one space on to the paired space. For self duality to be possible the field strength of the theory must be an element of a space that is mapped back into the same space by the Hodge operation. The usual ansatz is to truncate the theory, taking fields that live only in the first space. This was the case with the example given above and in the ansatz used in reference [4,5]. Fields that are in this space may couple to dyonic objets and hence are natural candidates for electromagnetic duality.

If we consider the sum of an element of one space with an element of its pair, then we can see that the Hodge operation will map such a sum of elements back onto the same space. That is the space formed by the direct sum of a space with its pair. So it is possible to have electromagnetic self duality for fields living in such a pair of spaces. Later we shall develop this idea showing using a suitable ansatz, that it is possible to form such pairs that transform non trivially under a duality transformation.

We shall begin our analysis with fields in the first space. It is necessary to have a basis of $H^{(d/2)}(K^d, \mathbf{Z})$ Let $\{\gamma_I\}$ be such a basis. $I = 1$ to $b^{(d/2)}(K^d)$ where $b^{(d/2)}(K^d)$ is the $d/2$ Betti number associated with the compact manifold M^d. A canonical basis is chosen such that the intersection matrix

$$Q_{IJ} = \int_{K^d} \gamma_I \wedge \gamma_J$$

is antidiagonal. The period matrix is:

$$G_{IJ} = \int_{K^d} \gamma_I \wedge^* \gamma_J$$

For definiteness we shall now go to the case of M^{10} compactified on $M^4 \times T^6$. So, the $\{\gamma_I\}$ are the basis of 3-forms in T^6 and $b^3(T^6) = 20$. As it is a torus, the three form basis may be written in terms of a product of the one form basis. The action in 10 dimension is:

$$S_{10} = \int_{M^{10}} dC \wedge H + \frac{1}{2} \int_{M^{10}} H \wedge^* H \tag{7}$$

with also the self duality equation:

$$i_v(H - dC) = 0 \tag{8}$$

The ansatz taken will be as follows:

$$H = \sum_{I=1}^{20} F^I \gamma_I$$

$$C = \sum_{I=1}^{20} A^I \gamma_I \tag{9}$$

where $F^I \in \Lambda^2(M^4)$ and $A^I \in \Lambda^1(M^4)$ We then proceed as before. Picking out a particular v and then decomposing the 3-form basis, $\{\gamma_I\}$ into parallel and perpendicular parts via the equations: $i_v\gamma_a \neq 0$ for γ_a in the space *parallel* to v and $i_v\gamma_i = 0$ for γ_i in the space *perpendicular* to v. From now on the indices a,b will indicate γ parallel and i,j will indicate γ perpendicular. This projection decomposes the 3 form basis into 10 parallel and 10 perpendicular basis 3-forms. Hence, when we substitute the ansatz (9) into the self duality equation (8) we arrive at 10 equations:

$$F^a = dA^a. \tag{10}$$

First compactify M^{10} on $M^4 \times T^6$ which involves substituting in the ansatz (9) into (7) and then doing the necessary integrals over T^6, (as before keeping only the zero modes). We arrive at a 4-dimensional action. We then substitute in the equations (10) derived from the self duality equation (8) for a particular choice of v. Giving the action:

$$S_4 = \int_{M^4} dA^a \wedge F^i Q_{ai} + \frac{1}{2} \int_{M^4} dA^a \wedge^* dA^b G_{ab} + 2F^i \wedge^* dA^a G_{ia} + F^i \wedge^* F^j G_{ij} \tag{11}$$

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As before we must now integrate out the auxilary fields, F^i. Again, the integrals will be Gaussian. Doing the integration gives the following action for an abelian gauge theory:

$$S_4 = \int dA^a \wedge dA^b \sigma_{ab} + dA^a \wedge^* dA^b \tau_{ab} \tag{12}$$

Where the coupling matrices τ and σ are given by:

$$\tau_{ab} = G_{ab} + Q_{ai} G^{ij} Q_{jb} - G_{ai} G^{ij} G_{jb}$$

$$\sigma_{ab} = Q_{ai} G^{ij} G_{ib} + G_{ai} G^{ij} Q_{jb} \tag{13}$$

The raised indices indicate the inverse matrix, such that $G^{ab} = G_{ab}^{-1}$ and not the *parallel* components of G_{IJ}^{-1}. Note that apart from the usual curvature squared term there is also a topological term that is a generalization of the theta term for a U(1) gauge theory. We will see how the above transforms under duality when we calculate the more general case discussed below.

Before, we noted that it may be possible to construct theories that are self dual that contain fields of different form rank (We mean the rank of the fields after compactification; before compactification it is clear that the fields must have the same rank). These fields will live in the sum of paired spaces discussed earlier. As such, the ansatz considered previously may be viewed as a degenerate case; in which the pair of the space is the space itself. We now move on to consider the case where we have a sum of fields that live in such a pair of spaces. Replace the ansatz (9) with the following:

$$H = \sum_I A^I \mu_I + B^I \nu_I$$

$$C = \sum_I a^I \mu_I + b^I \nu_I \tag{14}$$

where

$$A^I \in \Lambda^1(M^4) \quad B^I \in \Lambda^3(M^4) \quad a \in \Lambda^0(M^4) \quad b \in \Lambda^2(M^4)$$

and $\{\mu_I\}$ is the canonical basis of $H^4(T^6, \mathbf{Z})$ and $\{\nu_I\}$ is the canonical basis of $H^2(T^6, \mathbf{Z})$. Note $b^2(T^6) = b^4(T^6) = 15$. Hence, I=1..15. As before we construct the period matrices associated with both bases. Let G be the period matrix of the 4-form basis, $\{\mu_I\}$ and F be the period matrix of the 2-form basis $\{\nu_I\}$. There will also be an intersection matrix Q defined by: $Q_{IJ} = \int_{T^6} \mu_I \wedge \nu_J$ which in the canonical basis will be antidiagonal. We now define *parallel* and *perpendicular* bases for both the 2 and 4 forms as before. The indices a,b indicate parallel 2-form and i,j indicate perpendicular 2-form. $\tilde{a}, \tilde{b}$ denotes parallel 4-form and $\tilde{i}, \tilde{j}$ denotes perpendicular 4-forms. It can be seen for any given one form there are 5 parallel and 10 perpendicular 2-forms and 10 parallel, 5 perpendicular 4-forms. Hence substituting in the

ansatz (14) into the self duality equation (8) we have for a particular choice of v a set 15 equations:

$$A^{\tilde{a}} - da^{\tilde{a}} = 0$$

$$B^a - db^a = 0. \tag{15}$$

We compactify $S_{M^{10}}$ as before, performing the necessary T^6 integrals which introduce the period and intersection matrices defined above. Then substitute in the self duality equations (15) into the compactified action. After throwing away irrelevant total derivatives, we integrate out all auxillary fields, B^i and $A^{\tilde{i}}$. This leaves, the following 4-dimensional action:

$$S_4 = \int_{M^4} \frac{1}{2}[da^{\tilde{a}} \wedge^* da^{\tilde{b}} \tilde{\tau}_{\tilde{a}\tilde{b}} + db^a \wedge^* db^b \tau_{ab}] - da^{\tilde{a}} \wedge db^b \sigma_{\tilde{a}b}, \tag{16}$$

where we have the following couplings:

$$\tilde{\tau}_{\tilde{a}\tilde{b}} = G_{\tilde{a}\tilde{b}} + Q_{\tilde{a}i}F^{ij}Q_{j\tilde{b}} - G_{\tilde{a}\tilde{i}}G^{\tilde{i}\tilde{j}}G_{\tilde{j}\tilde{b}}$$

$$\tau_{ab} = F_{ab} + Q_{a\tilde{i}}G^{\tilde{i}\tilde{j}}Q_{\tilde{j}b} - F_{ai}F^{ij}F_{jb}$$

$$\sigma_{\tilde{a}b} = Q_{\tilde{a}i}F^{ij}F_{jb} + G_{\tilde{a}\tilde{i}}G^{\tilde{i}\tilde{j}}Q_{\tilde{j}b}. \tag{17}$$

The action contains the usual kinetic terms for scalar fields and 2nd rank tensor fields. There is also the topological term which is a generalization of the theta term that couples the scalar and two form fields.

Note that if $G = F$ then we have the same equation for the coupling as before, (13). This confirms that the previous case is a degenerate version of the more general situation in which we have a paired space. Also, one can easily check that this formula for the coupling of the abelian gauge theory in terms of the period matrix of the compactified space reproduces the simple T^2 result. In this instance the period matrix is two dimensional and so the perpendicular and parallel parts are one dimensional and hence no matrix inverses are involved.

Now we wish to construct the generators for the duality transformation for the theory described above. We follow the previous S-duality example, by choosing different directions for v and then determining how the theory changes. It is obvious from equations (16) and (17) that only the coupling matrices change when a different direction for v is choosen. Hence, the duality related theories will have the same form with only the couplings being different. This implies ofcourse that the equations of motion of the duality related theories will only differ by the value of the coupling matrices given in the action. The equations of motion for the action (16) are simply the free field equations for scalar, one form and two form fields. Also each field strength has the usual Bianchi identity. The topological coupling between

the scalar and two form fields will be transparent to the classical equations of motion but will play a role in the partition function (by analogy with the usual theta term).

To simplify the calculation we calculate the coupling matrices for a compact space that is a product of three orthogonal generic 2-tori, each with area one. Obviously, $b^1(T^6) = 6$, so there are six possible choices for v. Each choice will give a duality related theory.

The coupling constant matrices were calculated explicitly for each choice of v. These gave:

For $v = a1$ (corresponding to the a cycle of the 1st torus)

$$\tau = \frac{1}{({}^1\tau_{22})} diag({}^1\tau_{22}, {}^2\tau_{11}, {}^2\tau_{22}, {}^3\tau_{11}, {}^3\tau_{22}) \tag{18.a}$$

$$\tilde{\tau} = 1 \oplus \begin{pmatrix} {}^2\tau_{11}{}^3\tau_{11} & {}^2\tau_{12}{}^3\tau_{12} \\ {}^2\tau_{12}{}^3\tau_{12} & {}^2\tau_{22}{}^3\tau_{22} \end{pmatrix} \oplus \begin{pmatrix} {}^2\tau_{11}{}^3\tau_{22} & {}^2\tau_{12}{}^3\tau_{12} \\ {}^2\tau_{12}{}^3\tau_{12} & {}^2\tau_{22}{}^3\tau_{11} \end{pmatrix} \oplus 1 \oplus \frac{1}{({}^1\tau_{22})} diag({}^2\tau_{11}, {}^2\tau_{22}, {}^3\tau_{11}, {}^3\tau_{22}) \tag{18.b}$$

$$\sigma = \frac{{}^1\tau_{12}}{{}^1\tau_{22}} M \tag{18.c}$$

The direct sum refers to blockdiagonal decomposition of the matrices. M is a 5×10 matrix which has a 4×4 identity matrix in the last block and zeros elsewhere. ${}^a\tau_{ij}$ refers to the ijth element of the period matrix of the a th 2-torus. This essentially shows how the moduli of the tori combine to give the coupling constants for the 4 dimensional theory.

The coupling matrices calculated for different choices of v can be related to the above coupling matrices (calculated for v lying in the $a1$ direction), as follows:

For $v = b1$ (corressponding to the b cycle of the first torus)

$$ {}^1\tau_{22} \leftrightarrow {}^1\tau_{11} \tag{19.1}$$

For $v = a2$ (corresponding to the a cycle of the second torus)

$$ {}^2\tau_{ij} \leftrightarrow {}^1\tau_{ij} \tag{19.2}$$

For $v = b2$

$$ {}^1\tau_{11} \leftrightarrow {}^2\tau_{22} $$

$$ {}^1\tau_{12} \leftrightarrow {}^2\tau_{12} \tag{19.3}$$

For $v = a3$

$$ {}^1\tau_{ij} \leftrightarrow {}^3\tau_{ij} \tag{19.4}$$

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For $v = b3$

$$^1\tau_{11} \leftrightarrow {}^3\tau_{22}$$

$$^1\tau_{12} \leftrightarrow {}^3\tau_{12} \tag{19.5}$$

These form a set of duality generators that act on the couplings of the theory. Note, the transformation $\tau_{11} \leftrightarrow \tau_{22}$ is equivalent to the imaginary part of $\tau \leftarrow \frac{-1}{\tau}$. So the duality generators we have above correspond to a generalization of the coupling inversion duality generator of SL(2,Z).

The difference between the S-duality (coupling constant inversion) and the transformations we describe above is that part of the coupling constant matrix is left invariant by the duality transformation. In the terms of our scheme for calculating these generators, this is a result of having cycles cycles that contain both projection directions. These are left invariant by the duality transformation; cycles that contain neither are of course projected out and so do not appear. The cycles that contain one of the directions are those that are transformed under duality. Simple counting of the number of 2 and 4 cycles with these properties confirms this picture.

To determine the other generators it is neccessary to look at the topological coupling, σ and determine the generators that corresponds to the SL(2,Z), $\tau \to \tau + 1$ Following the same arguements that lead the theta term being invariant in the partition function under $\tau \to \tau + m$ (where m is integer), we conclude that $\sigma \to \sigma + m$ leaves the partition function invariant. In terms of τ this is equivalent to

$$^1\tau_{12} \to {}^1\tau_{12} + m\,{}^1\tau_{22} \tag{20.1}$$

(for v chosen to be the a direction of the 1st torus) and

$$^1\tau_{12} \to {}^1\tau_{12} + m\,{}^1\tau_{11} \tag{20.2}$$

(for v chosen to be in the b direction of the 1st torus).

The generators are obviously given by taking $m = 1$. Now, to calculate the group it is neccessary to determine how combinations of the generators act. Indeed it is clear that the set of generators presented here is not the minimal set. For example, (19.3) can be formed by composing (19.1) and (19.2). The minimal set will be given by (19.1), (19.2), (19.4) and (20.1). Examining the compositions of these generators we find that we find that they generate the group $SL(2, Z) \times SL(2, Z) \times SL(2, Z)$. Such a group is what would be naively expected given that the compact space was taken to be a product of three orthogonal (area=1) tori. For the most general 6-torus, the explicit calculation of the coupling matrices

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in terms of the modular parameters of the 6-torus would be far more complicated. For such a case we would expect the group to be the modular group of the six torus with a non linear realization in terms of the coupling matrices. What we wish to stress here is the possible introduction of fields that live in the *paired* space, scalar and antisymmetric fields, that have couplings that tranform under some duality group associated with a compact space.

Conclusions

We have investigated some aspects of duality induced by dimensional reduction of a self dual p-form theory. It has been shown that a T^6 dimensional reduction of a self dual 4-form may include scalar and antisymmetric tensor fields as well as 1-form fields. The coupling matrices for these theories have been calculated. It was shown that the form of the theory is left invariant but the couplings transform non trivially under duality. Indeed, we express the coupling matrices as functions of the moduli for the torus. The generators of the duality transformation have been explicitly calculated and the group formed by composing these generators is described, giving the expected modular group. The possibility of including scalar and antisymmetric tensor fields may have interest when considering massive gauge theories in 4-dimensions.

Acknowledgements

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